Model Predictive Control for Distributed Systems: Coordination Strategies & Structure

Yi ZHENG, Shaoyuan LI
School of Electronic Information and Electrical Engineering
Shanghai Jiao Tong University
Aug. 1st, 2015
Dynamic Systems Optimization Group
-----Institute of Automation, Shanghai Jiao Tong University

Team Member
• Pro. Shaoyuan Li
• Pro. Ning Li
• Associate Pro. Yi Zheng
• Associate Pro. Jing Wu
• Associate Pro. Dang Huang
• PhD: 14, Master: 21

Researching Fields
• Predictive Control for Large-Scale Systems
• Plant-wide optimization
• Fuzzy system
• Application of advance control
CONTENTS

- Background
- Coordination strategies
- System structure
- Applications
- Conclusion
Background

Technology Development → large scale complicated

Large-scale process industry systems

Chemical process  Power station  Sewage disposal

Characteristics:

• More units with inputs and outputs
• Wide spatial distribution
• Couplings (Energy, mass, information)
• Complicated models with constraints and multi-objects
Centralized and decentralized control systems

**DCS, PLC, Sensors, transmitter**

-from H. Kirrmann's PPT (ABB)
Background

Field bus, Intelligent instrument, Network
→ convert of control structure and mode

Network
Field bus
WSN
Smart device

Information exchange
Local control separately

Centralized control
Distributed control systems
**Hierarchical structure of the industry process control**

Algorithms
- PID
- MPC
- Robust
- Optimal
- Decoupling

Application
- SISO
- MIMO
- Dynamic optimization
- Steady optimization

Global steady optimization

Device1: local optimization (steady or dynamic optimization)
- High/low logic
- PID
- Lead/Lag
- PID
- SUM

Device2: local optimization (steady or dynamic optimization)
- Model Predictive control —— MPC

Device1: Decentralized PID

Device2: Decentralized PID

Hierarchical structure of the industry process control

From Qin’s PPT
Background

Large-scale system theory (1970’s)

Decentralized/ Hierarchical control:

Decentralized control

Hierarchical control
Background

Hierarchical Decentralized vs Distributed Model Predictive Control

Networked distributed control systems
Background

Distributed system

Optimization

Real time optimization

Steady economic optimization

Distributed Control

Time scale

Flexible
Background

Control problem of distributed large-scale system

90’ Large-scale system theory

2008.09 European FP7 STREP project

Hierarchical and Distributed Model Predictive Control of Large-Scale Systems

http://www.ict-hd-mpc.eu

IEEE TAC, Automatica, JPC

W. B. Dunbar, IEEE TAC, 2007, 52, 1249-1263;
R. Scattolini, JPC, 2009, 19, 723-731, Comp.&ChEng, 2013, 51, 21-41

Applications in different process industry

Reactor/separator system

By Al-Gherwi, JPC, 2011

Morosana, Energy &Building, 2011

hydro-power valley

J. Zárate Flórez, Mathem. & Comp., 2012
Background
COORDINATION STRATEGIES FOR DISTRIBUTED MODEL PREDICTIVE CONTROL
Problems

Distributed System

• Physically composed by many interacting “small-scaled” plants (Spatially distributed)
• Distributed control structure (communication with network)

Distributed MPC

• Good dynamic performance
• Constraints
• Practical

The performance of entire system under the control of DMPC framework is not as good as that under the control of centralized MPC

E-mail: yizheng@sjtu.edu.cn
Problems

Existing Strategies of DMPC
Cataloged by Cost Function

- Local cost optimization
  \[ \min_{\Delta U_{i,k}} J_i(k) \]
  \[ = \min_{\Delta U_{i,k}} \left( \sum_{l=0}^{N-1} \sum_{i} \left( \hat{x}_{i,k+l|k} + u_{i,k+l|k} \right)^T Q_i + \right. \]
  \[ \left. \cdot \hat{x}_{i,k+N|k}^T P_i \right) \]

X. Du, S. Li, Y. Xi, ACC, 2000

- Iterative \( \rightarrow \) Nash optimality
S. Li , Y. Zhang, Q. Zhu, Info. Sci. 2005

- Stabilized LCO-DMPC
M. Farina, R. Scattolini, Automatica, 2012
W.B. Dunbar, IEEE TAC, 2007

- Global cost optimization

- Impact-region cost optimization

E-mail: yizheng@sjtu.edu.cn
Problems

Existing Strategies of DMPC
Cataloged by Cost Function

- Local cost optimization
- Global cost optimization

\[
\min_{\Delta U_i(k)} \sum_{j \in P} J_j(k)
\]

\[
\min_{\Delta U_{i,k}} J_i(k)
\]

\[
= \min_{\Delta U_{i,k}} \left( \sum_{l=0}^{N-1} \left\| \hat{x}_{i,k+l|k} + u_{i,k+l|k} \right\|_Q + \left\| \hat{x}_{i,k+N|k} \right\|_P \right)
\]

Iterative \(\rightarrow\) Pareto optimality

Venket. A. IEEE TCST, 2008

- Impact-region cost optimization
Existing Strategies of DMPC

Cataloged by Cost Function

- **Local cost optimization**
- **Global cost optimization**
- **Impact-region cost optimization**

Mathematical Formulations:

\[
\min_{\Delta U_{i,k}} \sum_{j \in P_i} J_j(k)
\]
\[
\min_{\Delta U_{i,k}} J_i(k)
\]
\[
\min_{\Delta U_{i,k}} \left( \sum_{l=0}^{N-1} \left\| \hat{x}_{i,k+l|k} + u_{i,k+l|k} \right\|_{Q_i} + \left\| \hat{x}_{i,k+N|k} \right\|_{P_i} \right)
\]

**Stabilizing ICO-DMPC**

*(A general framework for linear systems)*

**S. Li, Y. Zheng, Z. Lin, IEEE ASE, 2015**
Problems

With the increasing of coordination degree

• In most cases: Global Performance $\uparrow$ $\sqrt{\quad}$

• Existing Methods: Network connectivity $\uparrow$ not expected

Design a DMPC which could increase coordination degree without any increasing of network connectivity?

Impact-Region Cost Optimization DMPC

Strategies:

- Optimizing the performance of impacted-region, increasing coordination degree
- Neighbor to neighbor communication
**ICO-DMPC**

**Formulation**

- **Local Sub-system Model**

\[
x_{i,k+1} = A_{ii}x_{i,k} + B_{ii}u_{i,k} + \sum_{i \in P_u} A_{ij}x_{j,k}
\]

**Assumption**: There is a static feedback \( u_i = k_i x \), such that the closed-loop linear system is asymptotically stable. *(\( u_i = k_i x \) dunbar 2007 IEEE TAC, Farina2012 Automatica)*

- **Predictive Model**

\[
f_{i,k+l|k} = x_{i,k+l|k} = A_{ii}^l x_{i,k} + \sum_{h=1}^{l} A_{ii}^{l-h} B_{ii} u_{i,k+h-1|k} + \sum_{j \in P_i} \sum_{h=1}^{l} A_{ii}^{l-h} A_{ij} x_{j,k+h-1|k}
\]

\((*)\)

- **Impaction of input to its downstream neighbor**

\[
\frac{\partial f_{i,k+l|k}}{\partial u_{j,k+h-1|k}} = \sum_{p=h+1}^{l} \frac{\partial f_{i,k+l|k}}{\partial x_{j,k+p-1|k}} \frac{\partial x_{j,k+p-1|k}}{\partial u_{j,k+h-1|k}} = \sum_{p=h+1}^{l} A_{ii}^{l-p} A_{ij} A_{jj}^{p-h} B_{jj}
\]
ICO-DMPC

Formulation

- Performance of each subsystem-based MPC

\[
J_I^I(k) = \sum_{l=1}^{N-1} \left( \left\| x_{i,k+l|k} \right\|^2_{Q_i,l} + \left\| \Delta u_{i,k+l-1|k} \right\|^2_{R_i,l} \right) + \sum_{j \in \mathcal{P}_i} \sum_{l=1}^{N-1} \left\| (\hat{x}_{j,k+l|k} + \omega_i S_{ji,k+l|k}) \right\|^2_{Q_j,l} + \left\| x_{i,k+N|k} \right\|_{P_i} + \sum_{j \in \mathcal{P}_i} \left\| x_{j,k+N|k-1} + \omega_i S_{ji,k+N|k} \right\|_{P_j}
\]

\[
S_{ji,k+l|k} = \sum_{h=1}^{l} \sum_{p=h+1}^{l} A_{ji}^{l-p} A_{ii}^p B_{ii} (u_{i,k+h-1|k} - \hat{u}_{i,k+h-1|k})
\]

\[
Q_i = \text{block} - \text{diag}\{Q_{i,1}, Q_{i,2}, \ldots, Q_{i,N}\} > 0 \quad A_{di}^T P_i A_{di} - P_i = -\hat{Q}_i
\]

\[
R_i = \text{block} - \text{diag}\{R_{i,1}, R_{i,2}, \ldots, R_{i,N}\} > 0 \quad \hat{Q}_i = Q_i + K_i^T R_i K_i
\]

**Assumption:**

\[
A_o^T P A_o + A_o^T P A_d + A_d^T P A_o < \Phi = 2:
\]
ICO-DMPC

Formulation

Optimization problem

\[
\min_{u_i,k:k+l-1|k} J^I_i(k)
\]

s.t.

\[\sum_{s=1}^{i} \alpha_{l-s} \| x_{i,k+s|k} - x_{i,k+s|k-1} \|_2 \leq \frac{\xi \kappa \epsilon}{2 \sqrt{m m_1}}, l = 1, 2, \ldots, N - 1;\]

\[\| x_{i,k+N|k} - x_{i,k+N|k-1} \|_{P_i} \leq \frac{\kappa \epsilon}{2 \sqrt{m}},\]

\[\| x_{i,k+l|k} \|_{P_i} - \| \tilde{x}_{i,k+l|k} \|_{P_i} \leq \frac{\epsilon}{\mu N \sqrt{m}}, l = 1, 2, \ldots, N;\]

\[u_{i,k+l-1|k} \in U_i;\]

\[x_{i,k+N|k} \in \Omega_i(\epsilon/2).\]

\[m_1 = \max \text{ number of elements in } P_{i2}^u g\]

\[\mathcal{R}_l = \max_{i2} \max_{j2} \left( \frac{1}{n} \max_{i2} \hat{A}_i^l A_{ij} \right)^T P_j \hat{A}_i^l A_{ij} \mathcal{E}^o\]

\[l = 0; 1; \mathcal{E}^o; N \mathcal{E}^o\]
ICO-DMPC
Dual-mode algorithm

Controller i

Check if $x \in \mathcal{E}$

1. $x_k$, $\hat{x}_{i,k+1:k+N\mid k}$, $\hat{u}_{j,k,k+1:k+N\mid k}$
2. No local feedback control is also ok!

No local feed-back control is also ok!
Assumptions
• There exists an initial feasible control for each subsystem

Feasibility

Theorem 1: suppose that above assumption hold, \( x(k_0) \in X \) and all constraint are satisfied. Then the control and state pair \((u^f_{i,k:k+N-1|k}, x^f_{i,k+1:k+N|k,i})\) is a feasible solution to ICO-DMPC at every update.
Analysis

Stability

• **Theorem 2**: suppose that above assumption hold, \( x(k_0) \in X \) and all constraints are satisfied, and the following parametric condition hold

\[
\frac{(N \cdot i \cdot 1)}{2} + \frac{1}{i} \cdot \frac{1}{2} < 0
\]

Then, by application of ICO-DMPC algorithm, the closed–loop system is asymptotically stabilized to the origin.

Proof: details Please see the paper :

Impacted-Region Optimization for Distributed Model Predictive Control Systems with Constraints
Simulation

Multi-zone Building Temperature Regulation System

Subsystem models:

\[ S_1 : \]
\[ x_{1,k+1} = 0.64x_{1,k} + 0.32u_{1,k} + 0.13x_{2,k} \]
\[ y_{1,k} = x_{1,k}, \]

\[ S_2 : \]
\[ x_{2,k+1} = 0.61x_{2,k} + 0.31u_{2,k} + 0.12x_{1,k} + 0.12x_{3,k} \]
\[ y_{2,k} = x_{2,k}, \]

\[ S_3 : \]
\[ x_{3,k+1} = 0.62x_{3,k} + 0.33u_{3,k} + 0.12x_{2,k} + 0.12x_{4,k} \]
\[ y_{3,k} = x_{3,k}, \]

\[ S_4 : \]
\[ x_{4,k+1} = 0.67x_{4,k} + 0.33u_{4,k} + 0.13x_{3,k} \]
\[ y_{4,k} = x_{4,k}, \]
Simulation

Comparison with LCO-DMPC, CDMPC

The evolution of the states under the centralized MPC, LCO-DMPC and ICO-DMPC
Simulation

Comparison with LCO-DMPC, CDMPC

State square errors under each controller

<table>
<thead>
<tr>
<th>Items</th>
<th>CMPC</th>
<th>ICO-DMPC</th>
<th>LCO-DMPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>0.0190</td>
<td>0.1219</td>
<td>6.1832</td>
</tr>
<tr>
<td>$S_2$</td>
<td>2.3190</td>
<td>3.0572</td>
<td>18.9352</td>
</tr>
<tr>
<td>$S_3$</td>
<td>5.8060</td>
<td>7.2366</td>
<td>35.5856</td>
</tr>
<tr>
<td>$S_4$</td>
<td>2.3648</td>
<td>3.1232</td>
<td>21.2535</td>
</tr>
<tr>
<td>Total</td>
<td>10.5088</td>
<td>13.5390</td>
<td>81.9575</td>
</tr>
</tbody>
</table>

\[
e_x - e_{CDMPC} \quad e_{CDMPC}
\]

ICO-DMPC:
3.04 (28.83%)

LCO-DMPC:
71.45 (679.89%)

Required network connectivity of each controller

<table>
<thead>
<tr>
<th>Items</th>
<th>CMPC</th>
<th>ICO-DMPC</th>
<th>LCO-DMPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>All</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$S_2$</td>
<td>All</td>
<td>1, 3</td>
<td>1, 3</td>
</tr>
<tr>
<td>$S_3$</td>
<td>All</td>
<td>2, 4</td>
<td>2, 4</td>
</tr>
<tr>
<td>$S_4$</td>
<td>All</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
SYSTEM STRUCTURE OF STABILIZED DISTRIBUTED MODEL PREDICTIVE CONTROL
Problem

Large-scale complicated distributed systems
Internet plus, cyber physics system, Industrial 4.0

Requires: Non global Information based asymptotically stabilized DMPC with constraints!

- Global cost optimization
  - Global information
  - Stabilizing
- Non-global information (LCO, ICO)
  - Non-global information based
  - Assumption strictly

2001- present difficult

Our Solution:
Control structure design
System Structure Design

Mass and energy coupling

Steam flow → System → Main pressure → Load

Burning coal

Different structures for different control purposes

Improve performance through reasonable structure design
System Structure Design

Structural characteristics of the information layer

Adjacency matrix

Knowledge Casual logic

- Interaction (Strong, weak);
- Controllability;
- Observability.

Ref: Theory of large-scale systems, Yugeng Xi
Control for complex system, A. Zeˇcevi´c & D.D. Šiljak.
System Structure Design

Process network model for large-scale system

Computers and Chemical Engineering 32 (2008) 1120-1134

- Easy performance analysis: closed-loop dissipativity condition
- Precondition: system structure design by the coupling relationship between mass and energy

Bond-graph of mass and energy balance
Structure of Stabilized DMPC

Global Information is **not** necessary!

- Iterative
- Communication once

Physical network

Information network

- GCO-DMPC
- MPC
- ICO-DMPC
System decomposition

Structure model

\[
\begin{bmatrix}
1 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 1 \\
\end{bmatrix}
\]

\[
x(k + 1) = Ax(k)
\]

\[
\overline{A}_{ij} = \begin{cases} 
1, & \text{if } i \text{ is affected by } j \\
0, & \text{if } i \text{ is not affected by } j 
\end{cases}
\]

Advantages:

- Qualitative research
- Simple expression and easy to get
- Logic calculation
- Uniform model for nonlinear systems
  (Taylor series approximate)

Adjacency matrix
System decomposition

Dynamic system structural model

\[
\begin{align*}
\mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\
\mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k)
\end{align*}
\]

Adjacency matrix for dynamic system

\[
\mathbf{A}_a = \begin{bmatrix}
\mathbf{x} & \mathbf{u} & \mathbf{y} \\
\mathbf{x} & \mathbf{A}^T & 0 & \mathbf{C}^T \\
\mathbf{u} & \mathbf{B}^T & 0 & 0 \\
\mathbf{y} & 0 & 0 & 0
\end{bmatrix}
\]

N-step Adjacency Matrix

\[
\mathbf{R} = (\mathbf{I} \cup \mathbf{A}_a)^{N-1} = \begin{bmatrix}
(\mathbf{A}^T + \mathbf{I})^{N-1} & 0 & \mathbf{A}^T + \mathbf{I}^{N-2} & \mathbf{C}^T \\
\mathbf{B}^T(\mathbf{A}^T + \mathbf{I})^{N-2} & \mathbf{I} & \mathbf{B}^T(\mathbf{A}^T + \mathbf{I})^{N-3} & \mathbf{C}^T \\
0 & 0 & \mathbf{I} & \mathbf{I}
\end{bmatrix}
\]
Controllability of subsystems

Structure Controllability

Condition 1

System is controllable, if and only if

\[ \text{gr}(\overline{G}_c) = n^2 \]

\[ \overline{G}_c = \begin{bmatrix}
    B & I & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\
    0 & -A & B & -I & 0 & \ldots & 0 & 0 \\
    0 & 0 & 0 & -A & 0 & \ldots & 0 & 0 \\
    \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
    0 & \ldots & \ldots & 0 & 0 & B & I & 0 \\
    0 & \ldots & \ldots & 0 & 0 & 0 & -A & B \\
\end{bmatrix} \]

Condition 2

System is controllable, if and only if
1. Input reachable
2. \[ \text{gr}(\begin{bmatrix} \overline{A} & \overline{B} \end{bmatrix}) = n \]

\[ \text{gr}(\overline{A}) = \max(\text{rank}(A)) \]

Is the maxim rank of numerical matrices related to the structural matrix
APPLICATIONS OF DISTRIBUTED MODEL PREDICTIVE CONTROL
Laminar Cooling Process

Control objective
- Cooling curve of strip
- Final temperature
- Average temperature

Manipulated variables
- Strip speed
- Water fluxes of jet
- State of cooling water jet

Heavy Plate Mill, Baoshan Iron & Steel Co. Ltd.
Laminar Cooling Process

Control structure

Applied result

• FT qualification rate: 60% → 85%
• Hit rate of finished strip: 80% → 95%
• Production: 180 → 210 ton/hour

(Zheng & Li, JPC, 2009)
Micro-Grid Energy Management System

Distributed generation
- Renewable energies (wind turbine, photovoltaic)
- Adjustable energy (diesel engine, gas turbine)

Storage
- Storage battery

Loads
- Shift-able loads
- Un-adjustable loads

Micro-grid Structure of Wuning Technical Park, Shanghai, China
Micro-Grid Energy Management System

Control strategy

Multi-objective DMPC
Economic, pollution, peak value

Initialization
Micro-Grid Energy Management System

Numeric results

Performance of Micro-grid with different methods

<table>
<thead>
<tr>
<th>Items</th>
<th>Original load</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Cost pu.Yuan/kw</td>
<td>19.967</td>
<td>16.996</td>
</tr>
<tr>
<td>Pollution pu.Yuan/kw</td>
<td>7.991</td>
<td>6.187</td>
</tr>
<tr>
<td>Peak pu.h</td>
<td>0.971</td>
<td>0.177</td>
</tr>
<tr>
<td>Smooth (pu.h)^2</td>
<td>0.186</td>
<td>0.134</td>
</tr>
<tr>
<td>Fitness</td>
<td>29.703</td>
<td>23.940</td>
</tr>
</tbody>
</table>


Conclusion & Future Work

Conclusion

• **DMPC algorithm** which could increase coordination degree without any increasing of network connectivity

• **Structure decomposition** which provide an implementation framework for large scale coupling systems

Future work

Non global communication based constraint DMPC with guaranteeing of stabilization?
Thanks for your attention!